

# Photoemission From A Two Electron Quantum Dot

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We consider photoemission from a two-electron quantum dot and find analytic expression for the cross-section. We show that the emission cross-section from the ground state as a function of the magnetic field has sharp discontinuities corresponding to the singlet-triplet transitions for low magnetic fields and the transitions between magic numbers  $(2J+1)$  for high magnetic field. We also find the corrections to the photoemission cross-section from a more realistic quantum dot having a finite thickness due to nonvanishing extent of electron wave function in the direction perpendicular to the plane in which 2-D dot electrons are usually confined.

Artificial atoms or quantum dots, which are essentially electrons confined in a two-dimensional region with a magnetic field in the third direction have been the subject of intense experimental and theoretical activity over the last few years [1]. The artificial "hydrogen atom" (single electron confined in a circular region by a harmonic potential with a magnetic field in the perpendicular direction) was solved for the eigenvalues and eigenfunctions over seventy years ago by Fock [2]. The levels were experimentally observed [3] more than fifty years later with the advent of quantum dots. The artificial "helium atom" (two electrons confined in a circular region by a harmonic potential with a magnetic field in the third direction) was tackled sixty years after the hydrogen atom. Maksym and Chakraborty [4] and Wagner, Merkt and Chaplik [5] worked out the energy levels and found an incredibly rich structure. The ground state can change its parity as the magnetic field is changed and there can be singlet-triplet transition. However, spectroscopic studies did not reveal the spectacular features because of an effect noted by these authors [4], [5]. Instead, they relied on certain thermodynamic

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measurements to support the energy level structure that they obtained. This "helium atom" problem was exactly solved a few years later by Dineykhon and Nazmitdinov [6], giving answers which agreed with the findings of Maksym and Cahkrabarty [4] and Wagner et al [5]. In this note, we wish to point out that photoelectric effect is an experiment that would probe the different transitions in the ground state energy as the magnetic field is varied. We will show discontinuities in the cross-section as a function of the magnetic field corresponding to the singlet-triplet transitions and find the corrections due to finite thickness by imposing a stronger harmonic confinement in the z-direction.

Considering two electrons in a circular dot with a magnetic field perpendicular to the circular region, we can write the hamiltonian,

$$H = \sum_{j=1}^2 \left[ -\frac{\hbar^2}{2m^*} \nabla_j^2 + \frac{\omega_c}{2} (-i\hbar \nabla_{\phi_j}) + \frac{1}{2} m^* \omega_c^2 \rho_j^2 \right] + \frac{e^2}{4\pi\epsilon\epsilon_0} \frac{1}{|\vec{\rho}_1 - \vec{\rho}_2|} \quad (1)$$

where  $\vec{\rho}_1$  and  $\vec{\rho}_2$  are the two dimensional position vectors of the two electrons,  $\omega_c$  is the cyclotron frequency,  $m^*$  is the effective mass of the electron in the semiconductor and  $\Omega^2 = (\omega_0^2 + \frac{\omega_c^2}{4})$ . Using the COM coordinate  $\vec{\rho}_c = \frac{1}{2}(\vec{\rho}_1 + \vec{\rho}_2)$  and the relative coordinate  $\vec{\rho}_{rel} = (\vec{\rho}_1 - \vec{\rho}_2)$ , the hamiltonian clearly splits into  $H = H_c + H_{rel}$ , where  $H_c$  depends only on the center of mass coordinates. This part of the hamiltonian is a purely single electron hamiltonian and can be exactly solved. The part  $H_{rel}$  on the other hand involves the important Coulomb repulsion and is responsible for the rich structure of the energy spectrum. Now if there is an external electric field of long wavelength (far infrared spectroscopy) imposed on the system, then the dot size being much smaller than the wavelength, there is no appreciable change in the electric field across the sample and hence the contribution to the hamiltonian is  $e\vec{E} \cdot (\vec{\rho}_1 + \vec{\rho}_2) \exp(i\omega t) = 2e\vec{E} \cdot \vec{\rho}_c \exp(i\omega t)$  where  $\vec{E}$  is a constant electric field. Thus, this perturbation does not couple to the  $\vec{\rho}_{rel}$ -dependent part and is completely blind to the rich structure coming from. Now, if photoelectric effect is what we are interested in, then the external electromagnetic field can be considered as coming from a vector potential  $\vec{A}^{ext} = \hat{n}A_0 \exp(i\omega t)$  and the perturbation hamiltonian  $\tilde{H}$  is  $\frac{\vec{p} \cdot \vec{A}^{ext}}{m^*}$  to the lowest order, which can be written as

$$\tilde{H} = \frac{A_0}{m^*} [\vec{p}_1 \cdot \hat{n} \exp(i\vec{k} \cdot \vec{\rho}_1) + \vec{p}_2 \cdot \hat{n} \exp(i\vec{k} \cdot \vec{\rho}_2)] \exp(-i\omega t) \quad (2)$$

and since the dipole approximation is no longer required (i.e. long wavelength condition is not imposed), we have the contribution to the ionization cross-section coming from both  $H_{\vec{\rho}_c}$  and  $H_{\vec{\rho}_{rel}}$ .

The photoemission cross-section involves the matrix element  $\int \psi_f^*(\rho_1, \rho_2) \tilde{H} \psi_i(\rho_1, \rho_2) d^2\rho_1 d^2\rho_2$ . The initial state is the ground state of the dot and can be written as  $\phi_1(\vec{\rho}_c) \psi_2(\vec{\rho}_{rel})$  while the final state corresponds to a free electron of wave number  $\vec{q}$  and a bound electron in the lowest energy state of a single electron quantum dot i.e.  $\psi_f(\vec{\rho}_1, \vec{\rho}_2) = \phi(\vec{\rho}_1) \exp(i\vec{q} \cdot \vec{\rho}_2)$ . It is this integral which is sensitive to the nature of initial state. As the initial state undergoes a singlet-triplet transition, the orbital parity of the spatial wave function changes and there is a sudden jump in the matrix element

and hence the cross-section as a function of the magnetic field will show jumps at the singlet-triplet transitions.

To establish the above result, the most important fact that we need to know is the two-electron wave function for the ground state. We have found an extremely accurate variational wave function for the ground state [7]. This wave function (normalized) is

$$\Psi(\vec{\rho}_c, \vec{\rho}_{rel}) = \frac{1}{2^{(\frac{|l|}{2}+1)}\pi\tilde{a}_H\beta\Gamma(|l|+1)} \left(\frac{\rho_{rel}}{\beta}\right)^{|l|} \exp\left(-\frac{\rho_c^2}{4\tilde{a}_H^2}\right) \exp\left(-\frac{\rho_{rel}^2}{4\beta^2}\right) \exp(-il\phi_{rel}) \quad (3)$$

where  $\tilde{a}_H^2 = \frac{\hbar}{4m^*\Omega}$  and  $\beta$  is a variational parameter fixed by the energy minimization condition

$$x^4 - \sqrt{2} \frac{\tilde{a}_H}{a^*} \frac{\Gamma(|l| + \frac{1}{2})}{\Gamma(|l| + 2)} x - 1 = 0 \quad (4)$$

where  $x = \frac{\beta}{2\tilde{a}_H}$  and  $a^* = \frac{4\pi\epsilon\epsilon_0\hbar^2}{m^*e^2}$  is the Bohr radius of the semiconductor material (energy spectrum is plotted in fig.1). The final state corresponds to one free electron, and one electron in the ground state of single electron quantum dot.

The matrix element for photoemission can now be written as

$$\langle f|\tilde{H}|i\rangle = \frac{e}{m^*} \sqrt{\frac{2\pi\hbar}{\omega}} \left[ \int \int \psi_{final}^*(\vec{\rho}_1, \vec{\rho}_2) \exp(i\vec{k} \cdot \vec{\rho}_1) (-i\hbar\hat{n} \cdot \nabla_1) \psi_{initial}(\vec{\rho}_1, \vec{\rho}_2) d\vec{\rho}_1 d\vec{\rho}_2 + (1 \longleftrightarrow 2) \right] \quad (5)$$

The two contributions to  $\langle f|\tilde{H}|i\rangle$  will be equal and hence we need to evaluate only one integral I which is

$$I = \frac{e}{m^*} \sqrt{\frac{2\pi\hbar}{\omega}} \frac{(i)^2 \hbar \hat{n} \cdot (\vec{k} - \vec{q})}{\sqrt{2^{(|l|+3)}\pi^3\tilde{a}_H^2\beta^2\Gamma(|l|+1)A}} \int \int \exp[i(\vec{k} - \vec{q}) \cdot (\vec{\rho}_1)] \exp\left[-\frac{\rho_1^2}{4\tilde{a}_H^2}\right] \exp\left[-\frac{\rho_c^2}{4\tilde{a}_H^2}\right] \times \exp\left[-\frac{\rho_{rel}^2}{4\beta^2} - il\phi_{rel}\right] \left(\frac{\rho_{rel}}{\beta}\right)^{|l|} d^2\rho_1 d^2\rho_2 \quad (6)$$

where  $a_H^2 = 2\tilde{a}_H^2$ . Now,

$$\exp\left[-\frac{\rho_c^2}{4\tilde{a}_H^2}\right] \exp\left[-\frac{\rho_{rel}^2}{4\beta^2} - il\phi_{rel}\right] \left(\frac{\rho_{rel}}{\beta}\right)^{|l|} = \exp\left[-(\rho_1^2 + \rho_2^2)\left(\frac{1}{16\tilde{a}_H^2} + \frac{1}{4\beta^2}\right)\right] \exp\left[-\rho_1\rho_2\left(\frac{1}{8\tilde{a}_H^2} - \frac{1}{2\beta^2}\right)\right] \times \cos(\phi_1 - \phi_2) \left(\frac{\rho_1 \exp[-i\phi_1] - \rho_2 \exp[-i\phi_2]}{\beta}\right)^{(|l|)} \quad (7)$$

After putting the expression in equation (7) in equation (6) and writing  $\vec{k} - \vec{q} = \vec{K}$ , we note that  $\vec{K} \cdot \vec{\rho}_1 = K\rho_1 \cos(\phi_1 - \eta)$ . Now the angular integrations in the above integral can be performed by using, as necessary, the following identities:

$$\begin{aligned} \exp[ib \cos(\phi_1 - \phi_2)] &= \sum_{m=-\infty}^{\infty} (i)^m J_m(b) \exp[im(\phi_1 - \phi_2)] \\ \exp[-c \cos(\phi_1 - \phi_2)] &= \sum_{m=-\infty}^{\infty} (i)^{2m} I_m(c) \exp[im(\phi_1 - \phi_2)] \\ \int_0^{2\pi} \frac{d\phi}{2\pi} \exp[i(m-n)\phi] &= \delta_{m,n} \end{aligned} \quad (8)$$

After carrying out the angular integrals with the help of binomial expansion

$$\left(\frac{\rho_1 \exp[-i\phi_1] - \rho_2 \exp[-i\phi_2]}{\beta}\right)^{|l|} = \sum_{t=0}^{|l|} \frac{\Gamma(|l|+1)}{\Gamma(t+1)\Gamma(|l|-t+1)} \frac{\rho_1^{|l|-t} \rho_2^t}{\beta^{|l|}} \exp[-i(|l|-t)\phi_1] \exp[-it\phi_2] \quad (9)$$

and writing the constant term outside as  $C_l$  we are left with radial integral parts

$$\begin{aligned} \langle f | \tilde{H} | i \rangle &= 2 \frac{C_l}{\beta^{|l|}} \int \rho_1 d\rho_1 \rho_2 d\rho_2 \sum_{t=0}^l C_t^{|l|} \rho_1^{|l|-t} \rho_2^t \exp[-\rho_1^2 b] \exp[-\rho_2^2 c] (-1)^t (i)^{|l|} \times \\ &\quad \exp[-i |l| \eta (2\pi)^2 J_l(K\rho_1) I_t(d\rho_1 \rho_2)] \end{aligned} \quad (10)$$

$$\begin{aligned} &= \frac{(2\pi)^2 C_l}{4\beta^{|l|} bc} \sum_{t=0}^l (-1)^t C_t^l (i)^l \exp[-il\eta] \frac{1}{(\sqrt{b})^{l-t} \sqrt{c}^t} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} (-1)^p \frac{\Gamma(p+q+|l|+1)}{\Gamma(p+1)\Gamma(q+1)\Gamma(|l|+p+1)} \times \\ &\quad \left(\frac{K}{2\sqrt{b}}\right)^{2p+|l|} \left(\frac{d}{2\sqrt{bc}}\right)^{2q+t} \end{aligned} \quad (11)$$

After some manipulations and using the generating function of associated Laguerre polynomial we find the closed form

$$\langle f | \tilde{H} | i \rangle = \frac{(2\pi)^2 C_l}{4\beta^{|l|} bc} \left(\frac{K}{2b}\right)^{|l|} \exp[-il\eta + i\frac{l\pi}{2}] \frac{(1 - \frac{d}{2c})^{|l|}}{(1 - \frac{d^2}{4bc})^{|l|+1}} \exp\left[-\frac{K^2}{4b(1 - \frac{d^2}{4bc})}\right] \quad (12)$$

where  $b = \frac{1+\frac{2}{x_H^2}}{8a_H^2}$ ,  $c = \frac{3+\frac{2}{x_H^2}}{8a_H^2}$  and  $d = \frac{1-\frac{2}{x_H^2}}{4a_H^2}$ . From this we get the expression for the differential cross-section

$$\frac{d\sigma}{d\phi_{\vec{q}}} = \frac{2^{8+3|l|}}{\Gamma(|l|+1)} \left[1 - (|l|+1) \frac{\Omega}{\omega} + \frac{l}{2} \frac{\omega_c}{\omega}\right] \sin^2 \theta \cos^2 \phi \frac{(x^2+2)^{2|l|} (Ka_H)^{2|l|}}{x^{2|l|-2} (x^2+6)^{2|l|+2}} \exp\left[-2 \frac{K^2 a_H^2 (3x^2+2)}{(6+x^2)}\right] \quad (13)$$

Here  $\vec{q}$  is the wave vector of the emitted electron,  $\vec{k}$  is the wave vector of the incident photon,  $\hbar\vec{K} = \hbar(\vec{k} - \vec{q})$  is the momentum transferred and  $\theta$  and  $\phi$  are respectively the angles  $\vec{q}$  makes with  $\vec{k}$  and  $\vec{k}\hat{n}$  plane where  $\hat{n}$  is the unit polarization vector of the incident photon.  $\sigma_0 = \frac{e^2}{ca^*2} \left(\frac{m_e}{m^*}\right)^2 \left(\frac{2\pi}{\hbar}\right)$  is a constant extracted to express the differential cross-section expression in a dimensionless form. So,

$$K^2 = k^2 + q^2 - 2kq \cos \theta \quad (14)$$

holds. and  $\cos \phi = \sin \theta_{\hat{n}} \cos(\phi_{\hat{n}} - \phi_{\vec{q}})$  and  $\cos \theta = \sin \theta_{\vec{k}} \cos(\phi_{\vec{k}} - \phi_{\vec{q}})$ . Therefore, it is very clear from the expression for the differential cross-section that it depends significantly on the direction of incidence and polarization. This for some simple cases can be illustrated easily. If  $\vec{k}$  is parallel to z-axis then  $\cos \theta = 0$  and  $\cos \phi = \cos \phi_{\vec{q}}$  or  $\cos \phi = \sin \phi_{\vec{q}}$  as  $\hat{n}$  is parallel to x or y-axis. So, the angular distribution is proportional to  $\cos^2 \phi_{\vec{q}}$  or  $\sin^2 \phi_{\vec{q}}$  and if it is the case of circular polarization then the angular distribution is proportional to  $(\cos^2 \phi_{\hat{n}} \cos^2 \phi_{\vec{q}} + \sin^2 \phi_{\hat{n}} \sin^2 \phi_{\vec{q}})$  and only if  $\hat{n} = \frac{(\hat{x} \pm i\hat{y})}{\sqrt{2}}$  then it becomes isotropic. But, when the  $\vec{k}$  lies in the x-y plane then with all the cases of circular polarization we shall have angular dependence. It also becomes apparent from the expression that emission count is larger in the direction of polarization compared to other cases and if the photon

is linearly polarized in the z-direction then there is no emission. So, depending on the 'l' values of the ground state as a function of magnetic field the cross-section would have different angular distribution as well as discontinuities characterizing transitions of the ground state. In this case it will also be found that if Based on these one can probe now these transitions experimentally. For this purpose one has to choose carefully the magnetic field strength, incident photon frequency and the abovementioned directions. We plot the dimensionless expression of the differential cross-section (fig.2) for the transitions  $l = 0 \rightarrow l = 1$  and  $l = 1 \rightarrow l = 2$  which take place (for our chosen system size  $\hbar\omega_0 = 4meV$ ) at 1.3T and 6.1T respectively when  $\vec{k} \parallel \hat{z}$  and  $\vec{q} \parallel \hat{n}$  (i.e. when it is maximum). From the plot it can be seen that there are discontinuities at the magnetic field strengths where singlet-triplet transitions are taking place and where for the cases of  $l = 0$  and  $l = 1$  the cross-section increases with magnetic field strength for  $l = 2$  it decreases. This can be understood on the physical ground in analogy with atomic photo-effect. As magnetic field strength increases wave function is compressed more and more that means electrons become more tightly bound and emission increases. When transition takes place depending on the energetics electron wave function becomes further compressed which manifests in the x-values for different 'l' values and the emission count shows a jump. But, after sufficient increase in the magnetic field strength the energy value also changes considerably and with it ionization energy changes. Thus upon keeping the incident frequency fixed after a certain range of magnetic field the cross-section decreases with increasing field strength.

In the real dots there is finite extent of the wave function in the z-direction and for the experiments it also needs to be considered. We take this into account by considering a stronger harmonic confinement in the z-direction. with this the hamiltonian gets modified with the term

$$H_z = -\frac{\hbar^2}{2m^*} \frac{d^2}{dz^2} + \frac{1}{2} m^* \omega_z^2 z^2 \quad (15)$$

and the Coulomb term becomes  $\frac{e^2}{4\pi\epsilon\epsilon_0} \frac{1}{|\vec{r}_1 - \vec{r}_2|}$ . With the  $H_z$  part we have the wavefunction modified by

$$\psi_3(z) = \frac{1}{\sqrt{2^n \Gamma(n_z + 1) \pi^{1/2} \lambda}} \exp(-\frac{z^2}{2\lambda^2}) H_n(\frac{z}{\lambda}) \quad (16)$$

and the energy is modified by the term  $E_z = (n_z + \frac{1}{2})\hbar\omega_z$ . When solved variationally (introducing two parameters  $\beta_1$  and  $\beta_2$  instead of two independent oscillator lengths) we have for the two-electron dot the following coupled characteristic equations

$$x^4 [1 - \frac{2}{y^3} (\frac{a_H}{a^*}) (\frac{a_H}{\lambda_{rel}})^3 \frac{\Gamma(|l| + 1)}{\Gamma(\frac{5}{2} + |l|)} {}_2F_1(\frac{3}{2}, 2 + |l|, \frac{5}{2} + |l|, 1 - 2(\frac{x}{y})^2 (\frac{a_H}{\lambda_{rel}})^2)] = 1 \quad (17)$$

$$y^4 - y (\frac{\lambda_{rel}}{a^*}) \frac{\Gamma(|l| + 1)}{\Gamma(\frac{3}{2} + |l|)} {}_2F_1(\frac{1}{2}, 1 + |l|, \frac{3}{2} + |l|, 1 - 2(\frac{x}{y})^2 (\frac{a_H}{\lambda_{rel}})^2) + 2 \frac{x^2}{y} \frac{a_H^2}{a^* \lambda_{rel}} \frac{\Gamma(|l| + 2)}{\Gamma(\frac{5}{2} + |l|)} \times \\ {}_2F_1(\frac{3}{2}, 2 + |l|, \frac{5}{2} + |l|, 1 - 2(\frac{x}{y})^2 (\frac{a_H}{\lambda_{rel}})^2) = 1 \quad (18)$$

where oscillator lengths are related as  $\lambda^2 = \frac{\lambda_{rel}^2}{2} = \frac{\hbar}{m^* \omega_z}$  and  $y = \frac{\beta_2}{\lambda_{rel}}$ . When solved numerically, from the energy spectrum it is found that transitions of the ground state are taking place at higher magnetic field values ( $l = 0 \rightarrow l = 1$  and  $l = 1 \rightarrow l = 2$  occur at 3.1T and 11.2T respectively for our chosen ratio  $\frac{\omega_z}{\omega_0} = 9$ ) as it becomes evident from fig.5 and the dimensionless differential cross-section now becomes

$$\begin{aligned} \frac{\frac{d\sigma}{d\Omega_{\vec{q}}}}{\sigma_0} &= \frac{2^{8+3|l|}}{\Gamma(|l|+1)} \left[ 1 - (|l|+1) \frac{\Omega}{\omega} + \frac{l}{2} \frac{\omega_c}{\omega} \right] \sin^2 \theta \cos^2 \phi \frac{(x^2+2)^{2|l|} (K_\rho a_H)^{2|l|}}{x^{2|l|-2} (x^2+6)^{2|l|+2}} \exp \left[ -2 \frac{K_\rho^2 a_H^2 (3x^2+2)}{(6+x^2)} \right] \times \\ &\quad \frac{8}{\sqrt{2\pi}} \frac{1}{y} \left( \frac{\lambda_{rel}}{a^*} \right) \frac{1}{\left( \frac{3}{y^4} + \frac{20}{y^2} + 8 \right)} \exp \left[ -K_z^2 \lambda_{rel}^2 \left( \frac{1}{\left( 3 + \frac{1}{y^2} \right)} + \frac{\left( 2 - \frac{1}{y^2} \right)^2}{4 \left( 3 + \frac{1}{y^2} \right) \left( 1 + \frac{1}{y^2} \right) - \left( 2 - \frac{1}{y^2} \right)^2} \right) \right] \end{aligned} \quad (19)$$

Here,  $K_\rho^2 = K_x^2 + K_y^2$  and  $K_z^2 = K^2 - K_\rho^2 = K^2 \cos^2 \theta_{\vec{K}}$  and

$$\begin{aligned} \cos \theta_{\vec{K}} &= \frac{k \cos \theta_{\vec{k}} - q \cos \theta_{\vec{q}}}{\sqrt{k^2 + q^2 - 2kq \cos \theta}} \\ \cos \theta &= \sin \theta_{\vec{k}} \sin \theta_{\vec{q}} \cos(\phi_{\vec{k}} - \phi_{\vec{q}}) + \cos \theta_{\vec{k}} \cos \theta_{\vec{q}} \\ \sin \theta \cos \phi &= \sin \theta_{\vec{k}} \sin \theta_{\vec{q}} \cos(\phi_{\vec{k}} - \phi_{\vec{q}}) + \cos \theta_{\vec{k}} \cos \theta_{\vec{q}} \end{aligned} \quad (20)$$

Putting these in the expression for differential cross-section and by explicit integration one can find the total cross-section. For the incidence direction parallel to z-axis and emission in the direction of polarization we show the plot for this modified differential cross-section (fig.7) with the earlier photon energy. From the plot it is seen that the cross-section value is now sufficiently suppressed compared to earlier case as energy of the corresponding states have significantly. Also, due to this reason where in the earlier 2-D case decrease in the cross-section as a function of field strength started for  $l = 2$  for the 3-D dot it started decreasing right from the  $l = 1$  state after certain amount of increase and at  $l = 1 \rightarrow l = 2$  transition the count decreased in contrast to increase in 2-D situation.

We conclude by considering the feasibility of an experiment which would detect the above effect. The first thing to note is that we want a coupling of relative co-ordinate to the external electromagnetic field. Consequently the wavelength of the radiation must be smaller than the size of the dot which can be of the order of 100 nanometers. This implies that we will be dealing with energetic photons and photo-emission will be from the bound state in the dot into vacuum. There will be emission from the semiconductor as well and we would have to subtract this background contribution. This can be achieved by studying the angular distribution. The distribution of the electron knocked out from the rest of the solid will be isotropic where as the distribution of the electrons coming from the dot would have a zero in the forward direction. This enables one to know the background which can now be subtracted to find the cross-section of the photoemission from the dot. Another point is that as the magnetic field applied to the dot can be considered to be local compared to the bulk, with the change of magnetic field while there will be change in the angular dependence and counts from the dot, that from the bulk would remain unchanged.

An alternative technique is to do the experiment first with the GaAs layer without dot. This determine the background. Next, one can repeat it with single-electron dots and finally with two-electron dots. The difference between the two-electron dot and the single-electron dot will exhibit the correlation effect.

From the plot for angular distribution for differential cross-section (fig.3, fig.4) it becomes clear that when transitions occur the angular distribution also changes and maximum of emission is obtained for a definite angle of incidence and this angle changes with transitions. So, at the time of experiments keeping the photon energy fixed and varying the angle of incidence and observing the change in angle corresponding to maxima due to change in 'l' states, the transitions can be probed. The most significant point which should be stressed is that for certain angle of incidence (as evident from fig.3 and fig.4) the count does not change even at the points of transitions. So to detect the transitions by the discontinuities in the cross-section, this angle should be carefully avoided. We have already argued above that if photon energy is kept fixed then all the transitions can not be observed properly. So, depending on the system size and field strength and 'l' values of the states angle of incidence and photon energy have to be carefully chosen and maximum of count will always be obtained in the direction of polarization.

Is the effect big enough to be measured? The scale for the cross-section of emission from the dot when compared to the scale for the emission from the bound state of an atom is the ratio  $\frac{m}{m^*} \frac{a_H^2}{\beta^2}$  which is much greater than unity and from our analytic expression for the emission cross-section it is evident that the ratio  $\sigma_0$  is a large number. So the scale for the emission cross-section is bigger when emission from the dot is involved. This should make the experiment quite feasible.

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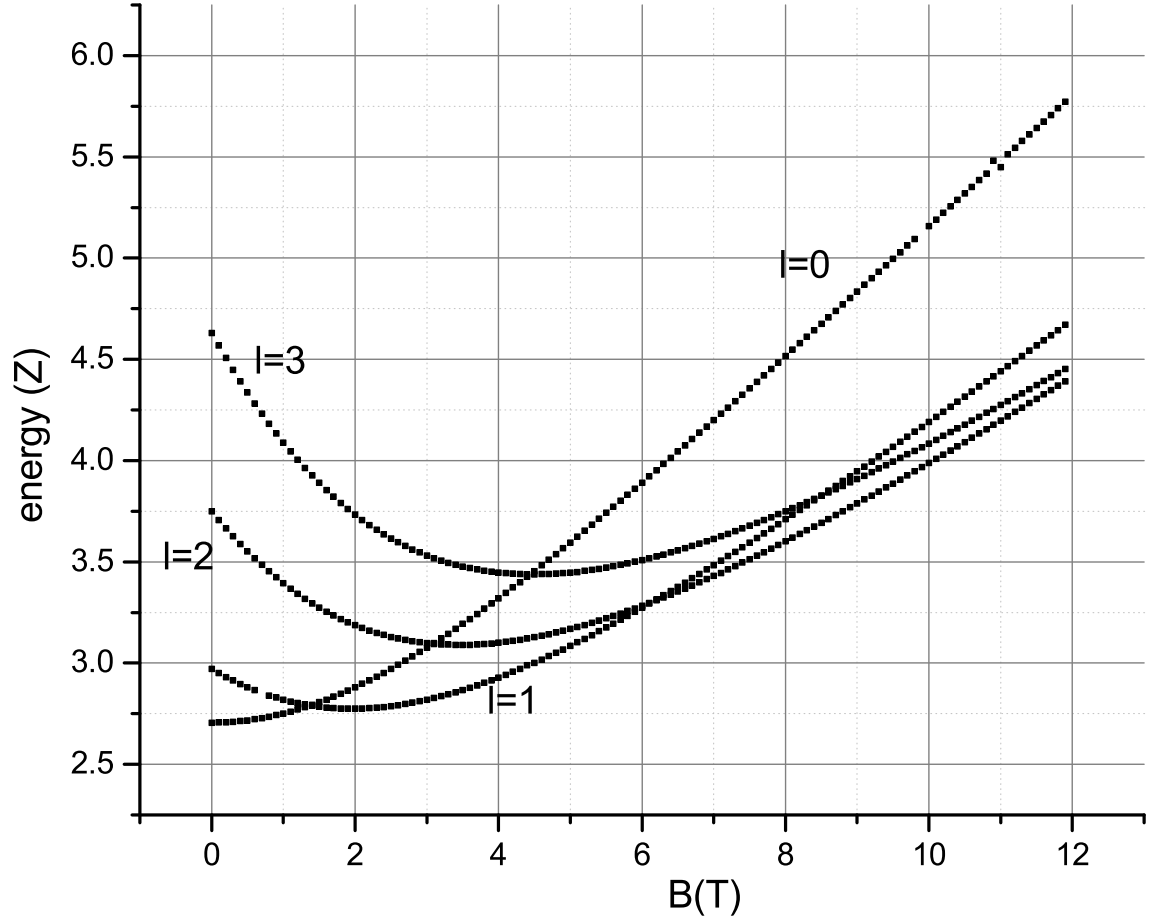


Figure 1:  $Z = \frac{E_{rel} + E_{spin}}{\hbar\omega_0}$  vs.  $B(T)$  is plotted for different 'l' values and  $l = 0 \rightarrow l = 1$ ,  $l = 1 \rightarrow l = 2$  transitions are taking place at  $B = 1.3T$  and  $B = 6.1T$  respectively and also other energy level crossings are present.



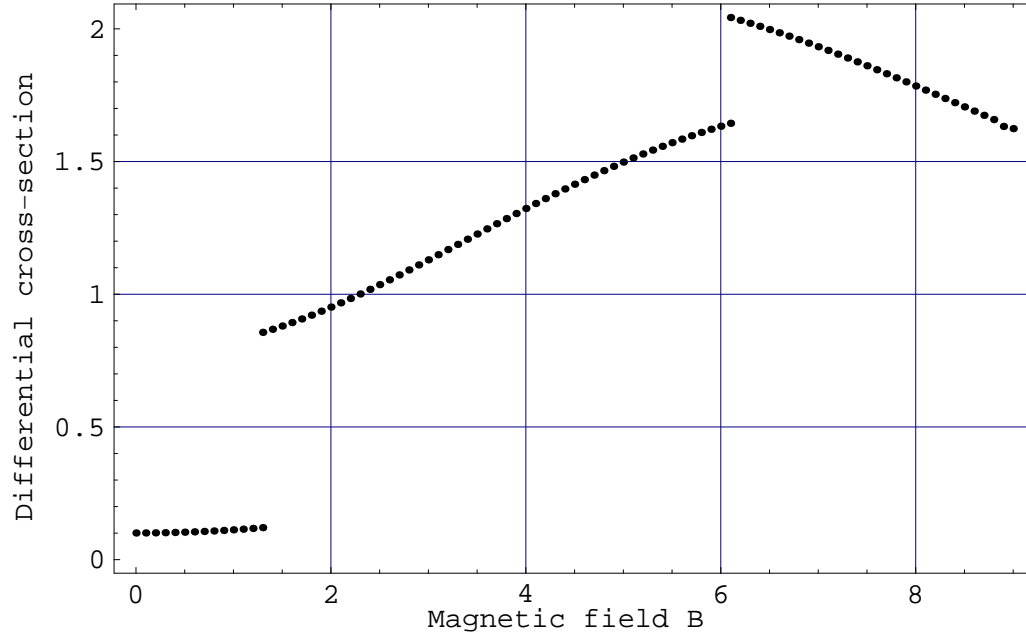


Figure 2:  $\frac{d\sigma}{d\phi d\vec{q}} \frac{\sigma_0}{\sigma_0}$  vs.  $B(T)$  is plotted for different 'l' values of the ground state as the B is varied and showing discontinuities as characteristic of the transitions. From the plot it is found that percentage change for  $l = 1 \rightarrow l = 2$  is  $\approx 4.5$  times smaller compared to  $l = 0 \rightarrow l = 1$  transition and also it is evident from the plot that at a fixed frequency behaviors of different 'l' cross-sections are going to change and for this reason both  $\omega$  and B have to be varied to observe all the transitions properly.

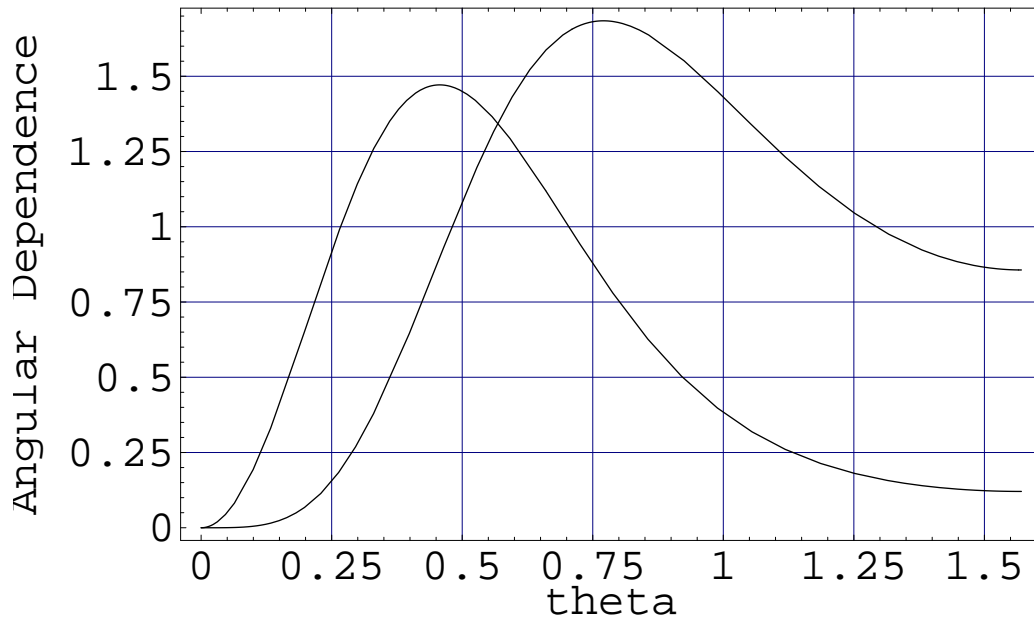


Figure 3: Difference in angular dependence of  $\frac{d\sigma}{d\phi \vec{q}}$  for transition  $l = 0 \rightarrow l = 1$  is shown

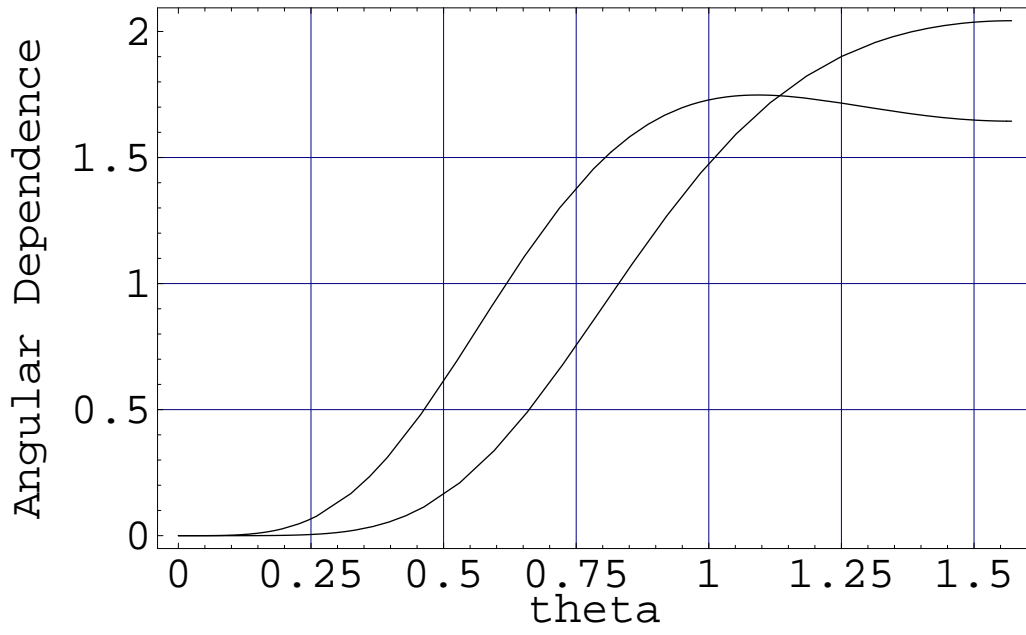


Figure 4: Difference in angular dependence of  $\frac{d\sigma}{d\phi \bar{q}} / \sigma_0$  for transition  $l = 1 \rightarrow l = 2$  is shown

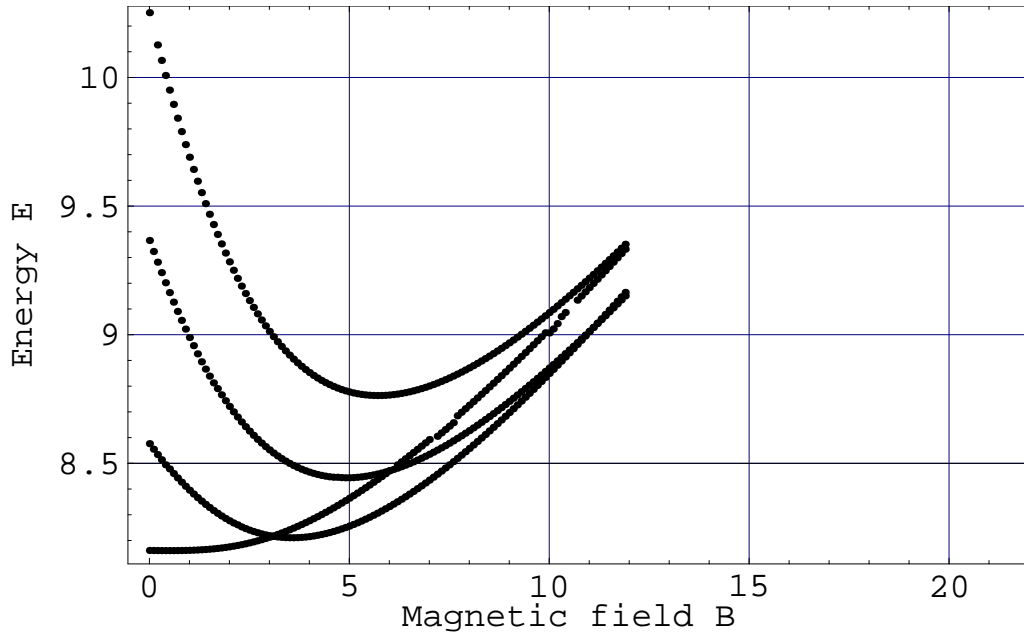


Figure 5:  $E = \frac{E_{rel} + E_{spin}}{\hbar\omega_0}$  vs.  $B(T)$  plotted for different 'l' values when finite thickness of the dot is taken into account. For this purpose  $\frac{\omega_z}{\omega_0} = 9$  has been taken.

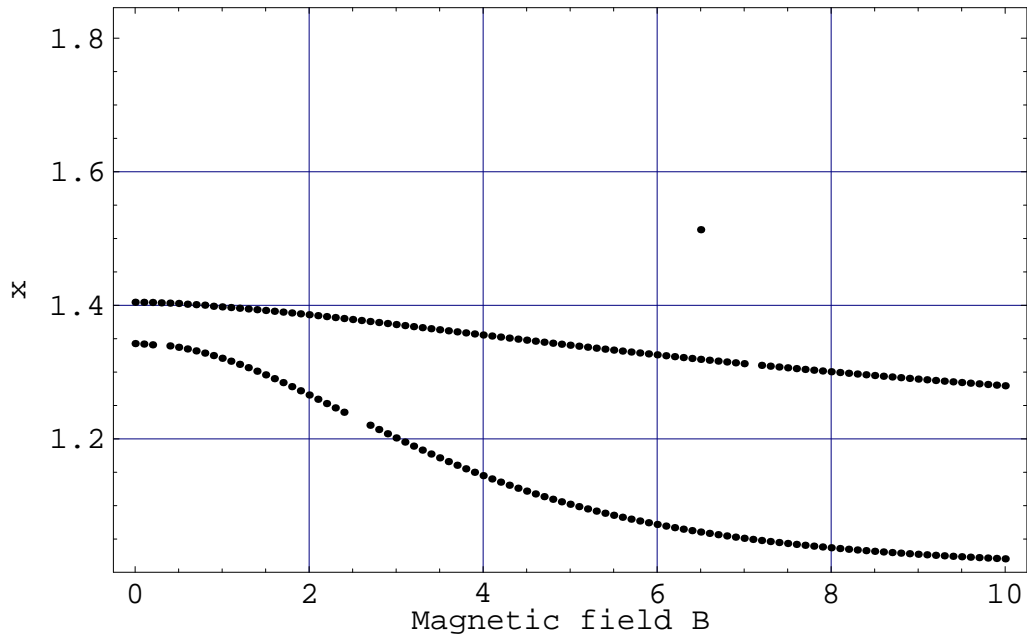


Figure 6:  $x$  vs.  $B(T)$  is plotted for  $l = 0$  for both the 2-D and realistic 3-D dot and it is clearly found that throughout the range of magnetic field  $x$  values have decreased for the later one.

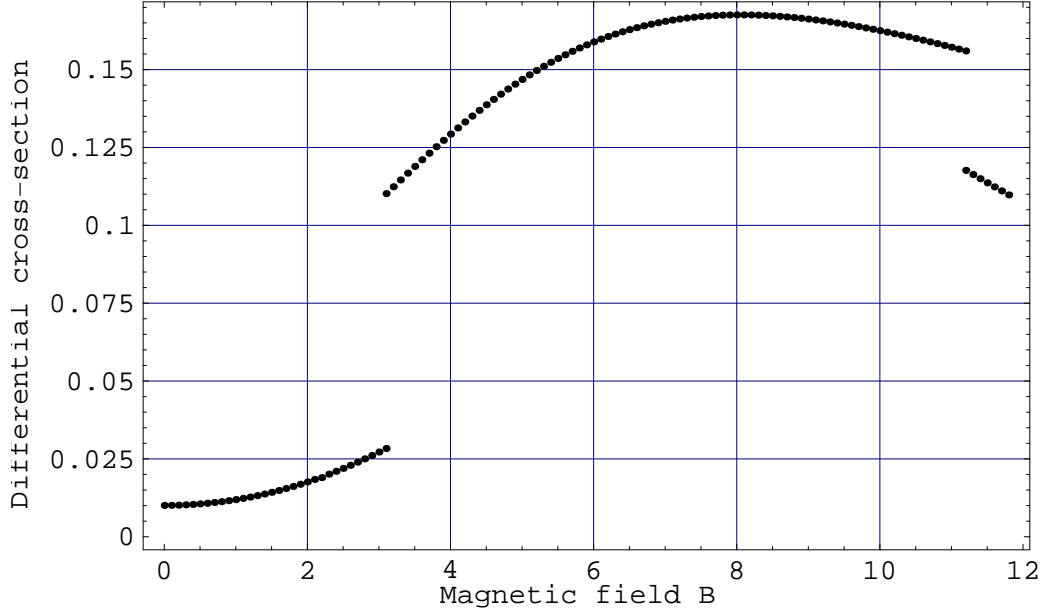


Figure 7:  $\frac{d\sigma}{d\Omega q}$  vs.  $B(T)$  is plotted for different 'l' values of the ground state as the B is varied and showing discontinuities as characteristic of the transitions. From the plot it is found that percentage change for  $l = 1 \rightarrow l = 2$  is  $\approx 4.5$  times smaller compared to  $l = 0 \rightarrow l = 1$  transition and also it is evident from the plot that at a fixed frequency behaviors of different 'l' cross-sections are going to change and for this reason both  $\omega$  and B have to be varied to observe all the transitions properly.